11/2

DISCUSSION 6

Mixed) Find the unique Nash.

$$\begin{array}{cc|c|c|c|}
 & & & P2 & \\
 & & L & C & R \\
\hline
 & T & \underline{4}, 1 & 1, \underline{9} & 3, 3 \\
P1 & m & 2, 0 & 2, \underline{6} & 2, 1 \\
 & B & 1, \underline{5} & \underline{4}, 0 & \underline{4}, 2 \\
\hline
\end{array}$$

↳ Remove strictly dominated? $(\frac{1}{2}T + \frac{1}{2}B)$ dominate m

$(\frac{1}{2}L + \frac{1}{2}C)$ dominate R.

⇓

$$\begin{array}{cc|c|c|c}
 & & P2 & & \\
 & & L & C & \\
\hline
P1 & T & \underline{4}, 1 & 1, \underline{9} & (p) \\
 & B & 1, \underline{5} & \underline{4}, 0 & (1-p) \\
\hline
 & & (q) & (1-q) &
\end{array}$$

↳ Play T: $4q + (1-q) = q + 4(1-q)$: Play B.

$3q + 1 = 4 - 3q$

$6q = 3$ $\quad q = 1/2$.

Play L: $p + 5(1-p) = 9p + 0(1-p)$: Play C

$5 - 4p = 9p$

$5 = 13p$

$5/13 = p$

↳ Equilibrium: $((5/13, 0, 8/13), (1/2, 1/2, 0))$.

T m B L C R

BOS23) $\pi_1 = (a - q_1 - \dots - q_n) q_1 - \frac{c}{2} q_1^2$

$\frac{d\pi_1}{dq_1} = 0.$ ⇒ $q_1 = \frac{a - q_2 - \dots - q_n}{c+2}$ (best response)

Symmetry ⇒ $q_1 = \frac{a - (n-1)q_1}{c+2}$ ⇒ $q_1 = \frac{a}{c+n+1}$ (Nash)

BCS24) $\frac{d\pi}{dx} = 0$ ⇒ $x = \frac{1-c-y}{2}$ (best response)

Symmetry ⇒ $x = y = \frac{1-c}{3}$ (Nash.)

Third price auction) a) Highest bid wins and pays third highest bid. $v_1 > v_2 > \dots > v_n > 0$. Ties split by higher bidder number. Find 2 symmetric Nash.

↳ $b_i = v_1$ for all i.

↳ $b_i = v_2$ for all i

↳ $b_i = v_3$? No P2 deviate.

b) Why is $b_i = v_i$ not Nash? P2 would deviate.